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Scalar and pseudoscalar QCD susceptibilities in nuclei

M. Ericson[1,2,a] and G. Chanfray[1]

[1] IPN Lyon, IN2P3-CNRS et UCB Lyon I, F-69622 Villeurbanne Cedex, France
[2] Theory division, CERN, CH-12111 Geneva, Switzerland

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Abstract. We study the staticscalar and pseudoscalar susceptibilities of QCD in the nuclear medium. We show that they become much closer than in the vacuum at normal nuclear matter density, a strong signal of the partial restoration of chiral symmetry.

PACS. 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes – 11.30.Rd Chiral symmetries – 12.40.Yx Hadron mass models and calculations – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.)

We focus in this paper on an aspect of chiral symmetry which has received little attention: the effect of the partial restoration on the susceptibilities of QCD, related to the fluctuations of the quark densities. In the phase of broken symmetry, there exist two susceptibilities. One along the spontaneous magnetization axis, and the second one along the perpendicular direction.

In QCD, the scalar susceptibility represents the modification of the order parameter, *i.e.*, of the quark condensate, to a small perturbation of the parameter responsible for the explicit breaking of the symmetry, which is the quark mass:

$$\chi_{\rm S} = \frac{\partial\langle\bar{q}q\rangle}{\partial m_q} = 2\int {\rm d}t'\,{\rm d}\mathbf{r}' G_{\rm R}(\mathbf{r}=0, t=0, \mathbf{r}', t'), \qquad (1)$$

where $G_{\rm R}$ is the retarded quark scalar correlator:

$$G_{\rm R}\big(\mathbf{r}, t, \mathbf{r}', t'\big) = \Theta(t-t')\big\langle -i\big[\bar{q}q(\mathbf{r}, t), \bar{q}q\big(\mathbf{r}', t'\big)\big]\big\rangle. \qquad (2)$$

The susceptibility represents space- and time-integrated correlators.

In the linear σ model the symmetry-breaking part of the Lagrangian is proportional to the σ field:

$$\mathcal{L}_{\chi{\rm SB}} = c\sigma \qquad (3)$$

with $c = f_\pi m_\pi^2$. This quantity plays the role of the symmetry-breaking Lagrangian of QCD:

$$\mathcal{L}_{\chi{\rm SB}}^{\rm QCD} = -2m_q\bar{q}q, \qquad (4)$$

which entails the following correspondence between the QCD and effective theory correlators:

$$\frac{\big\langle\bar{q}q(x)\bar{q}q(0)\big\rangle}{\big\langle\bar{q}q\big\rangle_{\rm vac}^2} = \frac{\big\langle\sigma(x)\sigma(0)\big\rangle}{f_\pi^2}, \qquad (5)$$

where $\langle qq\rangle_{\rm vac}$ is the vacuum value of the condensate. The fluctuations of the quark density are thus carried by the σ field, the chiral partner of the pion. The in-medium propagation of the σ in the energy domain near the two-pion threshold, has been the object of several investigations (see, *e.g.*, [1–3]). Here we will focus on the low-energy region, below the particle-hole excitation energies.

The scalar susceptibility is given by

$$\chi_{\rm S} = 2\frac{\langle\bar{q}q\rangle_{\rm vac}^2}{f_\pi^2}\int_0^\infty {\rm d}\omega\left(\frac{2}{\pi\omega}\right){\rm Im}\,D_{\rm SS}(\mathbf{q}=0,\omega)$$
$$= 2\frac{\langle\bar{q}q\rangle_{\rm vac}^2}{f_\pi^2}\,{\rm Re}\,D_{\rm SS}(\mathbf{q}=0,\omega=0), \qquad (6)$$

where $D_{\rm SS}(\mathbf{q},\omega)$ is the Fourier transform of the scalar correlator:

$$D_{\rm SS}(\mathbf{q},\omega) = \int {\rm d}t\,{\rm d}\mathbf{r}e^{i\omega t}e^{-i\mathbf{q}\cdot\mathbf{r}}$$
$$\times\big\langle -iT\big(\sigma(\mathbf{r},t)-\langle\sigma\rangle, \sigma(0)-\langle\sigma\rangle\big)\big\rangle. \qquad (7)$$

In a simple picture where the sharp σ mass is reduced in the nuclear medium, *i.e.* m_σ replaced by some dropped value, m_σ^* [2], the static correlator is $\exp(-m_\sigma^* r)/r$. As m_σ^* goes to zero at full chiral symmetry restoration, the fluctuations acquire an infinite range as for fluids near the critical temperature. In the work of Hatsuda *et al.* [2] the σ mass modification arises from the tadpole term. Here, we

[a] e-mail: Magda.Ericson@cern.ch

include the polarization of the nuclear medium by the σ field. The lowest excitations are the nuclear ones, where a nucleon-hole state is excited. At higher energy the σ transforms into two pions. When at least one pion is a nuclear one this process also produces an in-medium modification of the σ propagator. At present we have only included the nuclear states. The coupling of the σ to the nucleon-hole excitations modifies the scalar field propagator as follows:

$$D_{\rm SS} = D^0_{\rm S}\big(1 + D^0_{\rm S} g^2_{\rm S} \Pi_{\rm SS}\big). \tag{8}$$

Here $D^0_{\rm S} = 1/(q^2 - m^2_\sigma)$ is the bare σ propagator, $\Pi_{\rm SS}$ is the full scalar NN^{-1} polarization propagator and the quantity $g_{\rm S}$ is the σNN coupling constant. In the linear σ model, as well as in quantum hadrodynamics, this quantity has a value close to 10. Using this value, for a first estimate, and the free Fermi gas expression for the nucleon-hole propagator: ${\rm Re}\,\Pi^0(\mathbf{q} = \mathbf{0}, \omega = 0) \simeq -2M_N k_{\rm F}/\pi^2$ we get

$$\chi_{\rm S} \simeq \chi_{\rm S,vac}\left(1 + \frac{2g^2_{\rm S} M_N k_{\rm F}}{\pi^2 m^2_\sigma}\right). \tag{9}$$

Notice that the scalar susceptibility is, as the quark condensate, negative. This first-order correction leads to a large modification of the σ propagator at normal density: $D_{\rm S} \sim 12 D^0_{\rm S}$ at $\rho = \rho_0$ indicating a sizable in-medium increase of the scalar susceptibility. This effect can be seen as arising from an in-medium decrease of the σ mass. This mass is a screening mass and not the energy of the pole of the σ propagator at zero momentum.

Now, quantitatively, we have to know the full $\Pi_{\rm SS}(\mathbf{q} = 0, \omega = 0)$, which is a pure nuclear physics problem, on which experimental information can be obtained. Indeed, at low densities where non-relativistic effects are unimportant the scalar polarization propagator $\Pi_{\rm SS}$ is identical to the one for the ordinary density. It is then linked to the incompressibility factor K of nuclear matter, the magnitude of which is known experimentally

$$\frac{\Pi_{\rm SS}(\mathbf{q} = 0, \omega = 0)}{g^2_{\rm S}} = -\frac{9\rho_{\rm S}}{K}\,, \tag{10}$$

where $\rho_{\rm S}$ is the nucleon scalar density. With the currently suggested value, $K = 230\,{\rm MeV}$, which is practically the free Fermi gas one: $K = 3k^2_{\rm F}/M_N$, the quantity $\Pi_{\rm SS}(\mathbf{q} = 0, \omega = 0)$ also has the free Fermi gas value, which should validate the previous estimate.

However we have to discuss also in more detail the value of the σNN coupling constant entering the renormalization factor in eq. (8), which may depend on the density. This is the case in the quark-meson coupling model (QMC) of Guichon [4] which has a relevance for this problem. It is an extension of the quantum hadrodynamics model (QHD), but formulated at the quark level. In QMC the scalar and vector mesons couple directly to the quarks inside the nucleons, described by a bag model. The crucial ingredient of the model is the internal structure of the nucleon which adjusts to the presence of the scalar field. Under the influence of this attractive field the quark mass is lowered according to: $m^*_q = m_q - g^q_\sigma\langle\sigma\rangle$, where g^q_σ is the σ-quark coupling constant. Accordingly, the valence quark scalar number, which depends on the quark mass, also decreases, the quarks becoming more relativistic. This effect is directly related to the QCD scalar susceptibility of the bag, $\chi^{\rm bag}_{\rm S}$. Introducing the scalar charge, $Q_{\rm S}$, defined as the valence quark scalar number:

$$\begin{aligned} Q_{\rm S}(\langle\sigma\rangle) &= \int_{\rm bag} {\rm d}\mathbf{r}\big(\langle\bar q q(\mathbf{r})\rangle - \langle\bar q q\rangle_{\rm vac}\big) \\ &= Q_{\rm S}(m_q) + \chi^{\rm bag}_{\rm S}\big(m^*_q - m_q\big) \\ &= Q_{\rm S}(\langle\sigma\rangle = 0) - g^q_\sigma \chi^{\rm bag}_{\rm S}\langle\sigma\rangle. \end{aligned} \tag{11}$$

Now, the scalar charge acting as the source of the scalar field, a decrease of the scalar charge amounts to a lowering of the σNN coupling constant when the mean scalar field, *i.e.*, the density, increases:

$$g_{\rm S}(\rho) = g^q_\sigma Q_{\rm S}(\langle\sigma\rangle). \tag{12}$$

In QMC this mechanism is responsible for the saturation of nuclear matter. The introduction of $\chi^{\rm bag}_{\rm S}$, which is positive, amounts to an increase of the σ mass:

$$m^{*2}_\sigma(\rho) = m^2_\sigma + (g^q_\sigma)^2 \chi^{\rm bag}_{\rm S}\rho_{\rm S}, \tag{13}$$

which partly counteracts the decrease due to the mixing with NN^{-1} states. Note that the direct contribution of the bag susceptibility to the nuclear one is by itself small and effects mostly the mean scalar field. We use the values of Guichon *et al.* [5] for the density-dependent scalar coupling constant and also, for consistency, their value of the incompressibility: $K = 280\,{\rm MeV}$ (also compatible with the experimental allowed range). The enhancement factor of the scalar susceptibility is $\chi_{\rm S}(\rho_0)/\chi_{\rm S,vac} = 6.2$, still quite a large medium effect.

We now turn to the transverse susceptibility, linked to the fluctuations of the pseudoscalar quark density. We define it in such a way that it coincides with the scalar susceptibility in the restored phase:

$$\begin{aligned} \chi_{\rm PS} = 2 &\int {\rm d}t'\,{\rm d}\mathbf{r}'\Theta(t - t')x \\ &\times\left\langle -i\left[\bar q i\gamma_5\frac{\tau_\alpha}{2}q(0), \bar q i\gamma_5\frac{\tau_\alpha}{2}q(\mathbf{r}'t')\right]\right\rangle. \end{aligned} \tag{14}$$

This pseudoscalar susceptibility is related to the correlator of the divergence of the axial current since

$$\partial^\mu \mathcal{A}^\alpha_\mu(x) = 2m_q \bar q i\gamma_5\frac{\tau_\alpha}{2}q(x). \tag{15}$$

In the representations where PCAC holds the interpolating pion field is proportional to the divergence of the axial current according to

$$\partial^\mu \mathcal{A}^\alpha_\mu(x) = f_\pi m^2_\pi \Phi^\alpha(x). \tag{16}$$

The pseudoscalar susceptibility $\chi_{\rm PS}$ is then linked to the pion propagator, taken at zero momentum and energy:

$$\begin{aligned} \chi_{\rm PS} &= \frac{f^2_\pi m^4_\pi}{2m^2_q}\int {\rm d}t'\,{\rm d}\mathbf{r}'\Theta(t - t')\big\langle -i\big[\Phi^\alpha(0), \Phi^\alpha(\mathbf{r}'t')\big]\big\rangle \\ &= \frac{f^2_\pi m^4_\pi}{2m^2_q}\,{\rm Re}\,D_\pi(\mathbf{q} = 0, \omega = 0). \end{aligned} \tag{17}$$

Since the factor multiplying the pion propagator can be written as $2\langle\bar{q}q\rangle_{\rm vac}^2/f_\pi^2$, this equation is the analog of the one for the scalar susceptibility, with the pion replacing the σ. Thus in the linear σ model, where PCAC holds, the propagators of the two chiral partners, σ and π, govern the evolutions of the two susceptibilities. In the medium we denote $S(\mathbf{q},\omega)$ the pion self-energy so that

$$D_\pi(\mathbf{q}=0,\omega) = \left[\omega^2 - m_\pi^2 - S(\mathbf{q}=0,\omega)\right]^{-1}. \tag{18}$$

The expression of the self-energy depends on the representation. The one that should enter here is the one which applies to the PCAC representation. Its expression is not simple [6]. This complexity is, however, irrelevant for establishing a link with the condensate evolution. At zero four-momentum

$$\mathrm{Re}\, D_\pi(\mathbf{q}=0,\omega=0) = -\frac{1}{m_\pi^2 + S(0,0)}\,. \tag{19}$$

On the other hand, the evolution with density of the condensate is governed by the nuclear Σ commutator: Σ_A/A per nucleon, according to the exact expression:

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{(\Sigma_A/A)\rho}{f_\pi^2 m_\pi^2}\,, \tag{20}$$

which follows from the very definition of the nuclear Σ commutator as the expectation value over the nuclear ground state of the commutator between the axial charge and its time derivative. In the PCAC representation, the nuclear Σ commutator also represents the scattering amplitude for soft pions on the nuclear medium, $T(0,0)$ (per unit volume), with

$$\frac{(\Sigma_A/A)\rho}{f_\pi^2} = T(0,0). \tag{21}$$

This quantity is related to the pion self-energy, $S(q,\rho)$, which depends on the density, through

$$T(0,0) = \frac{S(0,0)}{1+S(0,0)/m_\pi^2}\,. \tag{22}$$

In this expression the denominator represents the coherent rescattering of the soft pion [7], which is needed in order to make the nuclear Σ commutator independent of the representation, ref. [6]. The link between the pseudoscalar susceptibility and the in-medium condensate is established by writing the condensate from its expression (eq. (20)) as

$$\frac{\langle\bar{q}q(\rho)\rangle}{\langle\bar{q}q\rangle_{\rm vac}} = 1 - \frac{T(0,0)}{m_\pi^2} = \frac{1}{1+S(0,0)/m_\pi^2}\,. \tag{23}$$

From the expression of the pseudoscalar susceptibility (eq. (17)) and using the GOR relation, one finally obtains the result

$$\chi_{\rm PS} = \frac{\langle\bar{q}q\rangle_{\rm vac}}{m_q}\frac{1}{1+S(0,0)/m_\pi^2} = \frac{\langle\bar{q}q(\rho)\rangle}{m_q}\,. \tag{24}$$

The pseudoscalar susceptibility follows the condensate evolution, *i.e.*, its magnitude decreases with density, with a linear dependence in the dilute limit where the relation $(\Sigma_A/A) = \Sigma_N$ holds. At normal density the susceptibility has thus decreased by 35%.

Our previous relation (24) between the transverse (pseudoscalar) susceptibility and the order parameter (the condensate) can be understood from the magnetic analogy. The rotational symmetry is intrinsically broken by a magnetic field $\mathbf{H}_0$ which aligns the spontaneous magnetization along its direction. The application of a small transverse field $\mathbf{H}_\perp$ rotates the magnetization $\mathbf{M}$ by an angle θ, such that it is now aligned in the direction of the resulting field $\mathbf{H}_0 + \mathbf{H}_\perp$. The transverse magnetization is $M_\perp = M\theta = M(H_\perp/H_0)$ and the transverse susceptibility is $\chi_\perp = M_\perp/H_\perp = M/H_0$, which is the analog of our formula (24).

Our results lead to the conclusion that at normal density the scalar and pseudoscalar susceptibilities become much closer than in the vacuum. As in the phase of restored symmetry they should become degenerate, the question is if this convergence is a signal of the partial symmetry restoration. The decrease in magnitude of the pseudoscalar one is clearly linked to the restoration since it follows the condensate. As for the scalar one which is the derivative of the order parameter with respect to the quark mass its evolution is also linked to the restoration process, which can be shown explicitly.

We have also explored how the convergence effect evolves at higher densities. However this extrapolation does not rely as previously on the experimental data of the incompressibility. For the scalar susceptibility we have taken the scalar coupling constant from ref. [5] and we have assumed that the polarization propagator keeps the free Fermi-gas expression with the effective nucleon mass of ref. [5]. The increase of the scalar susceptibility stabilizes or even decreases somewhat due to the fact that the nucleon reaction to the σ field manifests itself more with increasing density. However the influence of the two-pion continuum should also be included, which will be our next step, before a definite conclusion can be drawn. In summary we have studied the in-medium modifications of the two QCD susceptibilities, linked to the fluctuations of the scalar-isoscalar and the pseudoscalar-isovector quark densities. The first one is linked, through the linear σ model, to the propagator of the σ-meson, which in the nucleus mixes with the low-lying scalar-isoscalar nuclear excitations. At normal nuclear density this mixing produces an increase of the scalar susceptibility by a factor of about 6. This effect does not appear to increase further with increasing density. As for the pseudoscalar susceptibility, which is linked to the pion propagator, it follows the evolution of the condensate, *i.e.*, it decreases with density. The two combined effects make the scalar and pseudoscalar susceptibilities appreciably closer, at ρ_0, than in the vacuum. Their convergence is a clear, and amplified, signal of chiral symmetry restoration.

References

1. Z. Aouissat *et al.*, Nucl. Phys. A **581**, 471 (1995).
2. T. Hatsuda, T. Kunihiro, H. Shimizu, Phys. Rev. **82**, 2840 (1999).
3. J. Wambach, Z. Aouissat, P. Schuck, Nucl. Phys. A **690**, 127c (2001).
4. P.A.M. Guichon, Phys. Lett. B **200**, 235 (1988).
5. P.A.M. Guichon, K. Saito, E. Rodionov, A.W. Thomas, Nucl. Phys. A **601**, 349 (1996).
6. J. Delorme, G. Chanfray, M. Ericson, Nucl. Phys. A **603**, 239 (1996).
7. M. Ericson, Phys. Lett. B **101**, 11 (1993).